

Math 25
Precalculus
Problem Sets

Spring 2012

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1) Solve the equations.

a) $\frac{1}{3}x + \frac{2}{9} = 1$

b) $\sqrt{2x+3} = 5$

c) $(3w+1)^2 = 9$

d) $\frac{x+2}{3} = \frac{2}{x}$

e) Solve by factoring: $2m^2 - 10m + 8 = 0$

f) Use Quadratic Formula: $3k^2 + 11k + 9 = 0$

2) Solve the system of equations:
$$\begin{aligned} 2x + y &= 5 \\ -3x + 2y &= -1 \end{aligned}$$

3) Simplify the expressions.

a) $\frac{3 + \frac{2}{x}}{\frac{x}{4} + \frac{1}{8}}$

b) $\frac{1}{x+3} + \frac{2}{x-3}$

4) Find the equation of the line passing through (2,3) and (4, 9).

5) A poodle is shot out of a cannon at a circus.

The height of the dog after t seconds is given by $h = -16t^2 + 32t + 8$

a) Find the height of the dog after 1 second.

b) How long until the dog hits the ground?

c) Find the maximum height that the poodle will attain.

1) Evaluate the following expressions.

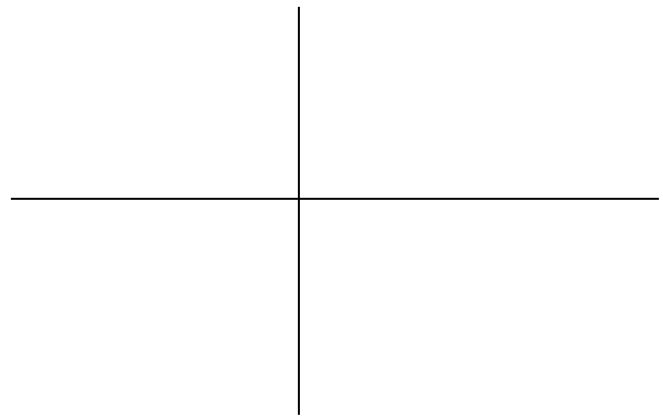
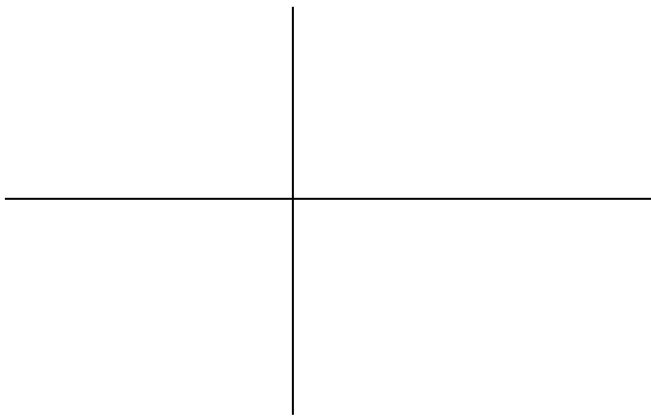
a)
$$\frac{5(6^2 - 1.2\sqrt{2.6^2 - 3(0.9)})}{e^4\sqrt{45}}$$

b)
$$\frac{\frac{44}{55} - \frac{55}{44}}{5.5^4 - 4.4^5}$$

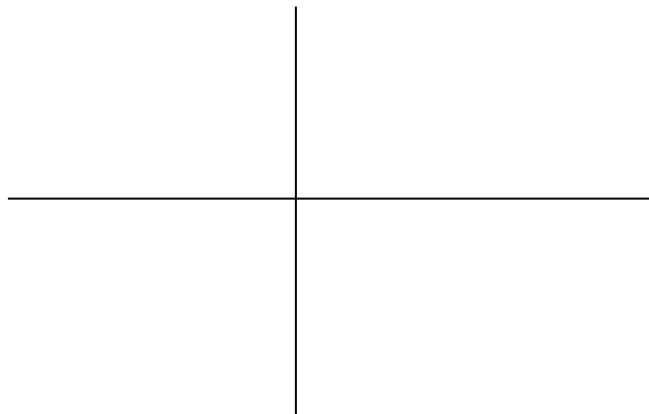
2) Use the graph to find the root(s) of the following functions. Draw the graph and label the x-intercepts.

a) $f(x) = -x^2 + 5x + 2$

b) $y = 5e^{-7x} - 12$



3) Find intersection points for graphs of $y_1 = x^2 - 3x + 5$ and $y_2 = 2x + 1$. Draw the graph and label the intersection points.



4) Create a table of 8 values for $f(x) = \frac{e^x}{x}$ near $x = 0$, using increments of 0.001. Use 4 positive and 4 negative values for x .

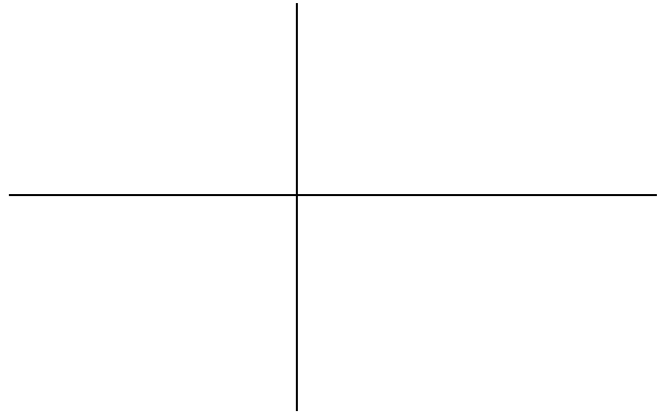
5) Graph the polynomial $y = 0.02x^3 - x^2 + 2x + 1$

a) Evaluate the function when $x = 30$.

b) Find all 3 x -intercepts for this polynomial.

c) Find the coordinates for the local maximum of the function.

d) Find the coordinates for the local minimum of the function.



Answer the following questions by looking at the graphs.

Do NOT put the graphs on this page.

1) Graph $y_1 = x^2$. Now compare this with the graphs of $y_2 = x^2 + 3$ and $y_3 = x^2 - 2$

Explain how the graphs of $f(x)$ and $f(x) + c$ compare. Be sure to explain what happens when c is positive and what happens when c is negative.

2) Graph $y_1 = x^2$. Now compare this with the graphs of $y_2 = 5x^2$ and $y_3 = 0.1x^2$

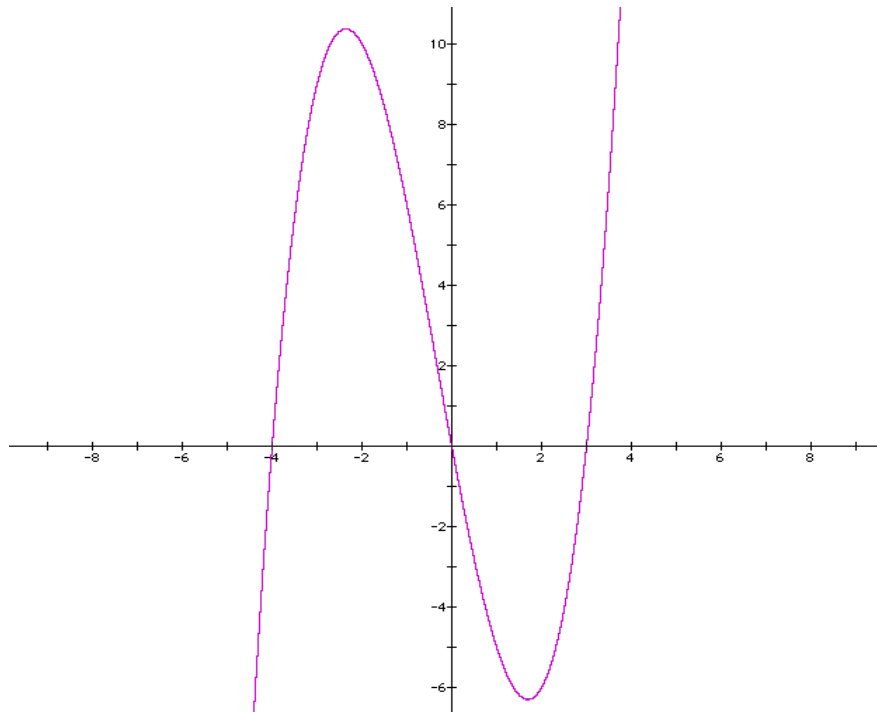
Explain how the graphs of $f(x)$ and $c \cdot f(x)$ compare.

3) Graph $y_1 = (x - 1)^2 + 2$. Now compare this with the graphs of $y_2 = (x - 4)^2 + 2$ and $y_3 = (x + 3)^2 + 2$

Explain how the graphs of $f(x - c)$ and $f(x + c)$ compare. Be sure to explain what happens when c is positive and what happens when c is negative.

4) Graph $y_1 = x^3 - 4x^2 + x - 1$. Now compare this with the graph of $y_2 = -(x^3 - 4x^2 + x - 1)$

What effect does multiplying a function by -1 have on the graph?



Draw each of the transformations of $f(x)$. Be sure to label key points!

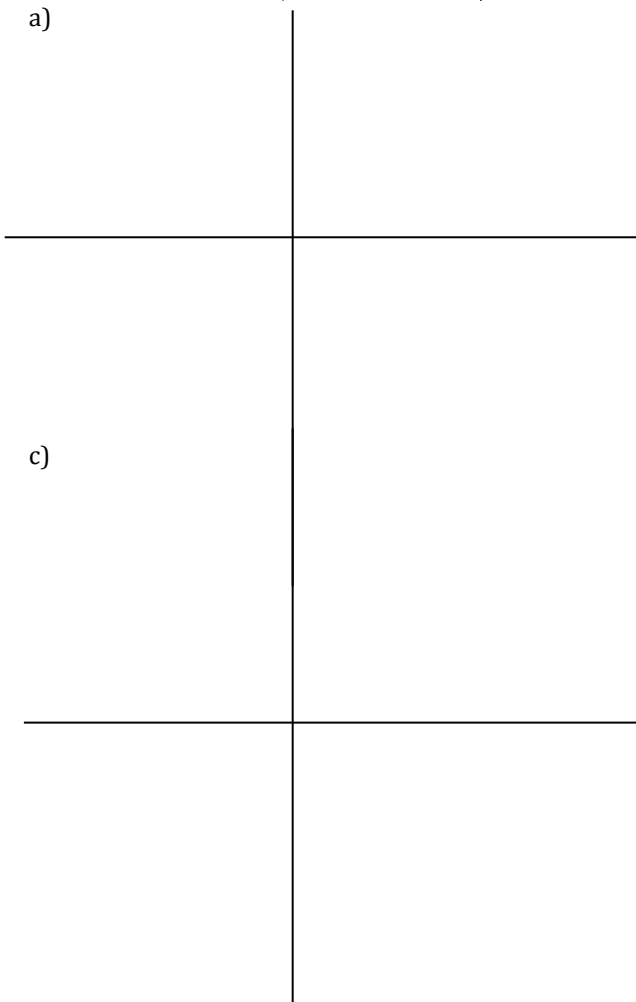
a) $0.5 \cdot f(x) - 2$

b) $f(x-1) + 3$

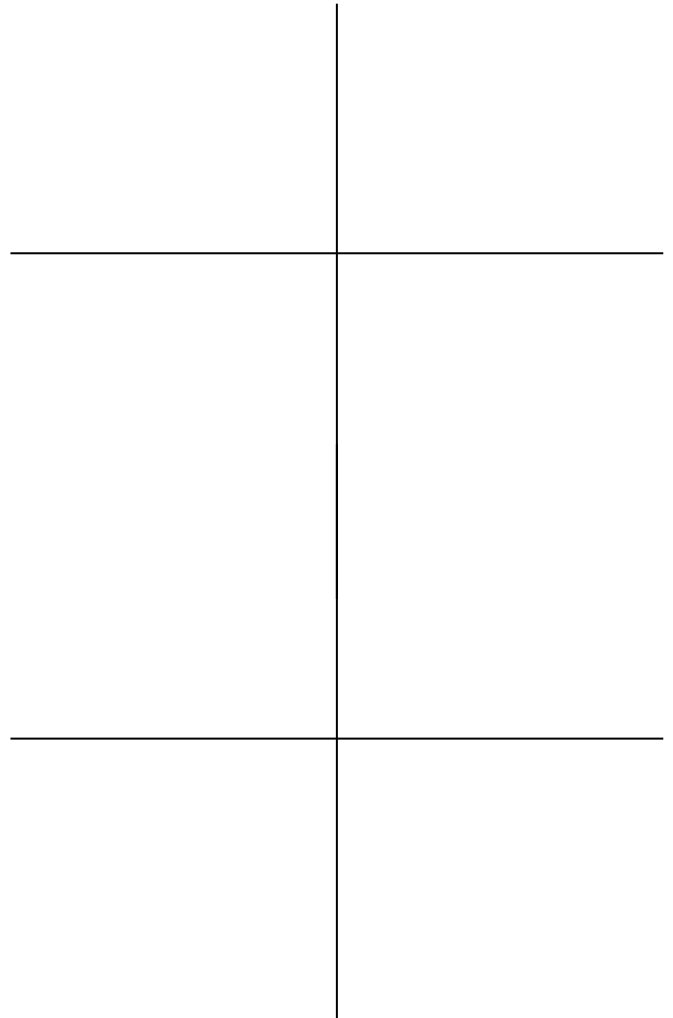
c) $-2 \cdot f(x)$

d) $-0.5f(x-1) + 2$

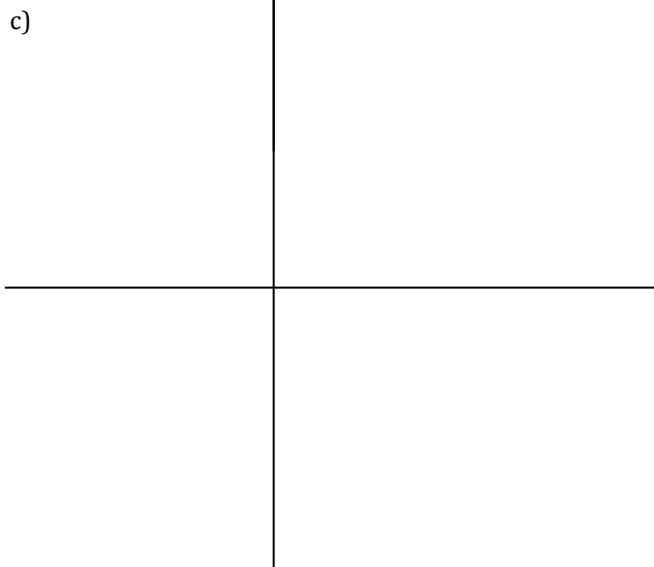
a)



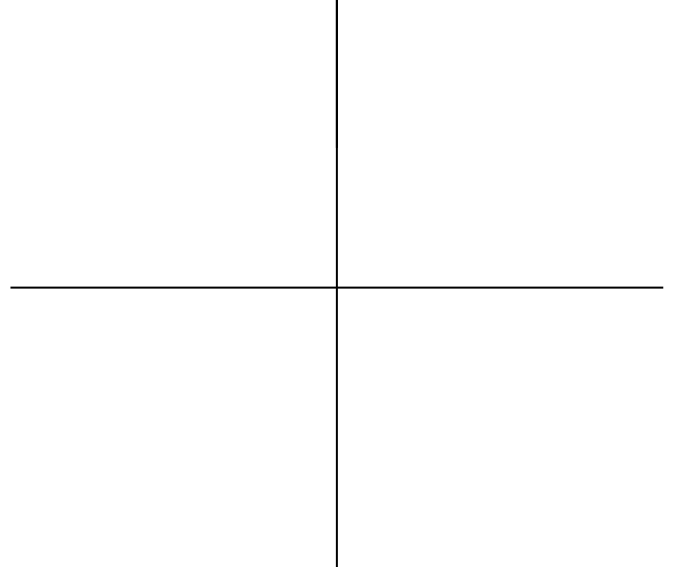
b)



c)



d)



1) Below are grades from last semester's Math 25 class. We will compare students grades on Problem Sets with their overall grade at the end of the semester. All scores are given as percentages.

Problem Sets	Overall Grade
27.1	36
48.7	60.1
70.5	68.8
79.7	79
84.2	80.2
89.2	90.5
90.8	93.2
76.5	77.1

a) Find the linear regression equation that relates Problem Sets and Overall Grades in this class.

b) Use the regression equation to predict the grade of a person that gets 50% on Problem Sets.

c) Use the regression equation to predict the grade a person needs on Problem Sets in order to get a B in this class.

d) What do parts a-c indicate about Problem Sets in this class?

2) In 1998, the population in Oakville was 61,000. In 2008, the population had grown to 96,700. Assume that the population grows at a constant rate.

a) How much does the population grow each year?

b) Find the equation of the line that give the population of Oakville in the year x .

c) When will the population exceed 125,000? Give the year and the month.

3) Ellie rented a car on two separate weekends. The first weekend it cost her \$64.06 and she drove 58 miles. The next weekend she drove 141 miles and it cost her \$69.87.

a) Graph the given data using x for miles and y for cost. Draw a line connecting the two data points.



b) Find the equation of the line from part a.

c) Explain what the slope of the line represents in this situation.

d) Find the y -intercept and explain what it represents in this situation.

4) A private school is hosting a fundraising dinner where the tickets are \$20 each. The school must rent supplies for \$1800 and the dinner costs them \$6 per plate. They hope to raise at least \$50,000 from the dinner.

a) Write an expression for the amount of money they will make.

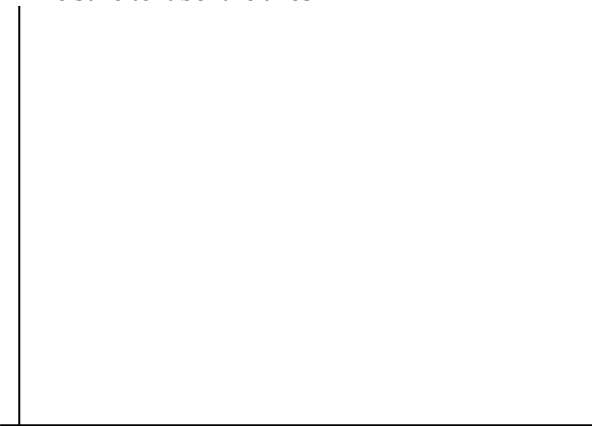
b) How many dinner tickets do they need to sell in order to reach their goal?

1) Let $f(x) = 2x^2 - 20x + 42$

a) Find all x-intercepts.

b) Find the midpoint of the x-intercepts.

c) Use the midpoint to find the vertex of the parabola.

d) What does $x = \frac{-b}{2a}$ tell you about the parabola?2) At the circus, a poodle is shot out of a canon. The height, in feet, of the dog after t seconds is give by $h(t) = -16t^2 + 78t + 6$.a) Graph the height function.
Be sure to label the axes.

b) What is the domain of this function?

c) What is the range of this function?

d) How long will the poodle be air born? Set up an equation and solve it algebraically.

3) A new movie theater holds 300 people. They found that when the price was \$5 per ticket, the average sales would be 270 people per show. Each time they increase the price by \$1, the average attendance drops by 15 people.
Let x be the price for a ticket.

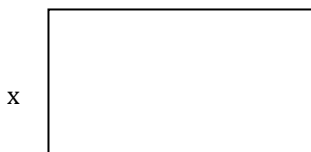
a) Find an expression, using x , for the number of people that will go to each show.

b) Find the function $R(x)$ that gives the amount of revenue the theater will make.

c) What price should they charge in order to maximize revenue?

4) A man just got a pig as a new pet. He wants to build a rectangular pigpen out of steel on two sides and aluminum on the other two sides. (Don't ask me why.) The steel fencing costs \$18 per linear foot and the aluminum is \$13 per linear foot. The pig needs 200 square feet of space to be comfortable.

a) Let x be the length of one side. Find a function for the cost of the pigpen in terms of x .



b) Find the dimensions of the pigpen that will minimize cost. Use a graphing calculator to find this.

5) The volume of a soda can is 12 ounces (or 355 cm^3). Find a function of x that gives the surface area of the can.

1) Find all of the roots of $f(x) = 8x^3 + 10x^2 - 63x$ **by factoring**.

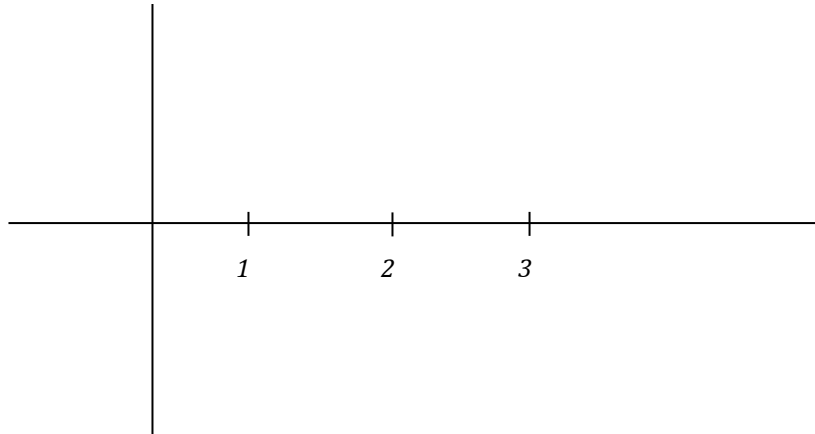
2) Use the fact that $x = 1$ is a root of $f(x) = x^3 + 3x^2 + 3x - 7$ to find all 3 roots of $f(x)$.

3) Use the fact that $x^2 - 9$ is a factor of $f(x) = x^4 + 5x^3 - 16x^2 - 45x + 63$ to find all 4 roots of $f(x)$.

4) Use the fact that $x^2 + 9$ is a factor of $f(x) = 2x^3 + 5x^2 + 18x + 45$ to find all 3 roots of $f(x)$.

5) Compare the graphs of the following functions $y_1 = (x + 2)(x - 2)$ and $y_2 = (x + 2)(x - 2)^2$ and $y_3 = (x + 2)(x - 2)^3$. Notice that all 3 functions have the same roots, yet $x = 2$ is a *repeated root* of y_2 and y_3 . How does having a repeated root affect the graph of a polynomial?

6) Based on the results of #5, what would the graph of $y = (x - 1)(x - 2)^2(x - 3)^3$ look like?



7) Create a polynomial function of degree 4 that has roots at 1, 2 and 3. Multiply it out.

1) Suppose that you were to drive from Salinas to Las Vegas. That trip involves 449 miles of driving in CA where the speed limit is 65 and 64 miles of driving in Nevada where the speed limit is 75.

a) Suppose that you planned to drive x mph above the posted speed limit the whole way.

Find the total time, $T(x)$, for the trip as a function of x . (Hint $d = r \cdot t$)

Add the two fractions to write your answer as a rational function.

b) Find $T(4)$ and explain it's meaning in this situation.

c) Solve $T(x) = 4$ and explain it's meaning in this situation.

d) Find the vertical asymptotes for $T(x)$ and explain what they represent in this situation.

e) Find the end behavior of $T(x)$ and explain what they represent in this situation.

2) Suppose that a polynomial has degree 5.

a) What is the least number of real roots it can have? Explain.

b) What is the most number of real roots it can have? Explain.

3) Start with the polynomial function $f(x) = 2x^3 + 5x^2 + 18x + 45$

a) It is a fact that $3i$ is a root of $f(x)$. Find the other complex root.

b) Find the quadratic factor of $f(x)$ that gives you the two complex roots of $f(x)$.

c) Divide $f(x)$ by the quadratic factor found in part b. Find the only real zero of $f(x)$.

4) Find ALL asymptotes for the functions below.

a) $y = \frac{3x^2 + 2x + 3}{x^2 - 4}$

b) $y = \frac{x^3 - 2x^2 + 3x - 1}{12x^2 + 5x - 7}$

- 3) The population of the United States is 309,690,000 and is growing exponentially at a rate of 2.1%.
- Find an equation for the population of the US in terms of time, t .
 - During what year will the US have over 330 million people?
 - How large will the US be in 2020?
- 4) Suppose that you have \$1200 to invest in a mutual fund that pays interest monthly.
- What would the rate have to be if the account was worth \$1800 in just 6 months?
 - How long would you have to leave the money in the account if the fund paid 17% and you wanted to double your money?
 - Suppose the fund paid 17% compounded monthly. What would be the corresponding **annual** rate?

1) Dioxin is a drug used to treat heart disease. At $t=0$, there was 0.5mg administered and each day the amount of drug in the bloodstream was measured. Below, x represents days and y represents the amount of dioxin in the bloodstream.

x	0	1	2	3	4	5	6	7	8
y	0.5	0.345	0.238	0.164	0.113	0.078	0.054	0.037	0.026

a) Use exponential regression to find a function for the amount of Dioxin in the person's system after t days.

b) Find the amount Dioxin in the person's system after 3.5 hours.

c) It is necessary to keep 0.2 mg of Dioxin in a person's system to be effective. How often should the drug be administered?

2) A radioactive dye is given to a patient to help with an X-ray procedure. The amount of radioactivity, y , is given after t minutes.

t	0	1	2	3	4	5	6	7	8	9	10
y	10,023	8174	6693	5500	4489	3683	3061	2479	2045	1645	1326

a) Use exponential regression to find a function for the amount of radioactivity in the person's system after t minutes.

b) Find the amount of radiation in the person's system after one day.

c) In order to be effective, the radiation level needs to be at least 3100. How long does the hospital have to take the X-ray?

3) Forensic scientists use Newton's Law of Cooling to determine how long a person has been dead based on the temperature of the body. $T(t) = T_s + (98.6 - T_s)e^{-0.1947t}$ gives the temperature, T , after t hours for a body where the surrounding temperature is T_s . Suppose that a person was found dead in a room that was 50° .

a) Find the temperature of the body after 3 hours.

b) How long has the person been dead if the body's temperature is 65° ?

4) The concentration of a drug in a person's system is given by the equation $A = A_0e^{-0.125t}$ where A_0 is the amount ingested at time $t = 0$ and A is the amount present after t hours.

a) If 50 mg of a certain drug was taken at 4PM, how much is present at 9:30PM?

b) A certain medication is useless if only 20% of the original amount is present in the patient's system. How often should the drug be taken?

For each problem tell me what each variable represents, set up a system of equations, then solve.

1) Farmer Ted has ducks and cows. The only problem is that Ted can't remember how many of each animal he has. He does remember that there are a total of 24 heads and 76 feet. How many of each animal does farmer Ted have?

2) Billy-Bob won \$1200 at the horse races. He decided to invest the money in two accounts, one that pays 8% interest and one that pays 15% interest annually. After one year he had made \$155.50 in interest. How much did Billy-Bob invest in each account?

3) Juan Valdez manages a gourmet coffee shop. He sells Honduran Dark Roast for \$9.25 per pound and Organic Costa Rican coffee for \$12.75 per pound. How much of each type would he need to use in order to make 50 pounds of coffee that sells for \$10 per pound?

4) The Rotary Club is trying to raise \$50,000 by selling tickets to a benefit concert. Children's tickets cost \$15 each, while adult tickets sell for \$50. How many tickets of each kind should they sell if there are 1400 seats in all?

5) Find the equation of the parabola that passes through the points: $(-1,13)$, $(1,1)$ and $(2,10)$

6) Osman rode his bike to his friend Elroy's house, which was 15 miles away. After he had been riding for half an hour, Osman got a flat tire and had to walk the rest of the way. The trip took him a total of 3 hours. If he rides the bike 4 times faster than he walks, how fast did Osman ride?

1) One national park is famous for their elk population. In 1970, there were 900 elk and each year there are 900 new elk, plus 40% of the ones from the previous year.

a) Find a_1 to a_5 , the first five terms of the sequence.

b) Will the elk population ever exceed 2000? Prove your answer.

2) Irma just started a new job. Her starting pay is \$50,000 a year and she gets a 5% raise each year.

a) How much will Irma make during her 20th year at this job?

b) In total, how much would Irma make over those 20 years?

3) A ball that is dropped from 12 feet bounces to half its previous height with each bounce.

a) Write the sequence of the first ten heights. Put your answer in inches.

b) Create a formula for this sequence.

4) Sal owes Vinnie \$8,343 as a result of a gambling debt. Vinnie is not too good with numbers. He agrees to let Sal pay half of the remaining debt each month.

a) How long until Sal has paid off his debt?

b) How much has Sal paid after the 12th payment?

5) On January 1 of each year, Anne invests \$500 in a retirement account that pays 7% interest compounded annually.

a) Find a_1 to a_5 , the first five terms of the sequence.

b) Find the formula for a_n

c) How long until the account is worth \$100,000?

- 1) Halley's comet has an orbit that is elliptical with the sun as a focus. Its equation is $\frac{x^2}{18.09^2} + \frac{y^2}{4.56^2} = 1$ where a and b are given in astronomical units (AU). Find the closest that the comet gets to the sun. (HINT: The sun is a focus.)

- 2) The equation $x^2 = -16400(y - 4100)$ gives the path of a satellite as it escapes Earth's gravity.
- a) Solve for y .

b) Graph the path of the satellite. Put Earth at the origin.

c) How close does the satellite come from hitting Earth?

3) Two 30-foot telephone poles are 100 feet apart. The cable that hangs between them hangs down to a height of 20 feet from the ground. Assuming that the shape of the cable is a parabola, find the equation of the parabola.

4) An oval running track is 100 yards across at its widest point and 50 yards across at its narrowest. Find the equation that gives the shape of this track.

5) Prove that the two branches of a hyperbola are NOT parabolas.

