Math 3B

Spring 2012

Problem Sets

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Math 3B Problem Set 1 Review Name ____________________

1) Find the first derivative, $\frac{dy}{dx}$.

a) $y = \sin^{-1}(x^2)$

b) $y = \ln(\tan x)$

c) $x \sec y + ye^{2x} = x$

d) $y = \dfrac{x^2 + 2x + 9}{x^8 + 5x}$

e) $y = x^x$

f) $y = (2x+3)^2(5x^2 + 9x - 1)^3$

2) Find the antiderivatives, if possible. Otherwise write, “can’t do yet.”

a) $\int \frac{4x+2}{x^2+x}\ dx$

b) $\int \sin^2 x\ dx$

c) $\int \frac{\sin^2 x}{\cos^3 x}\ dx$

d) $\int x\sqrt{2x+1}\ dx$

e) $\int x\sqrt{x^2+1}\ dx$

f) $\int \sqrt{x^2+1}\ dx$

Math 3B Problem Set 2: Integration Name____________________

Work on a separate page to find the antiderivatives. ATTACH THIS WORK. Write only the answers on this page. Write "no solution" for any that can't be done.

1. a) $\int e^{x^2}\,dx$ b) $\int xe^{x^2}\,dx$ c) $\int x^2e^{x^2}\,dx$

2. a) $\int \frac{2}{\sqrt{9-x^2}}\,dx$ b) $\int \frac{2x}{\sqrt{9-x^2}}\,dx$ c) $\int \frac{2x^2}{\sqrt{9-x^2}}\,dx$

3. a) $\int e^{\frac{1}{t}}\,dt$ b) $\int \frac{e^{\frac{1}{t}}}{t}\,dt$ c) $\int \frac{e^{\frac{1}{t}}}{t^2}\,dt$

4. a) $\int \sin x \cos x\,dx$ b) $\int \sin^2 x \cos x\,dx$ c) $\int \sin x \cos^2 x\,dx$

5. a) $\int \sin(2x)\,dx$ b) $\int x\sin(2x)\,dx$ c) $\int x^2\sin(2x)\,dx$

6. a) $\int \ln x\,dx$ b) $\int x\ln x\,dx$ c) $\int \frac{\ln x}{x}\,dx$

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Math 3B Problem Set 3 Differential Equations Name___________________

1) Suppose that a flashlight were shown in your face and then was moved away at a constant rate of 2 feet per second. It is a fact that the rate at which the intensity, I, of the light decreases is inversely proportional to cube of the distance, R, from the source of the light times the rate at which the distance increases. When the light is 10 feet from your face, the intensity is changing by 5 lumens per second.

a) Set up the differential equation and find the constant of proportionality, *k*.

b) Use the differential equation to find the distance the light is from your face if the light's intensity is decreasing at 1 lumen per second. (This is how we can find the distance from Earth to far off stars.)

c) Find the intensity of the light after 22 seconds.

d) How long until the intensity has decreased to 1 lumin?

2) A rock is dropped into a calm pond of water. This sends out ripples (concentric circles) from the source.

a) Find the orthogonal trajectories for the ripples.

b) Explain the physical meaning of the orthogonal trajectories in this situation.

3) Look at the graphs of $y = 5\sin x$ and $y = \sin(5x)$ on $[0,2\pi]$.

Are these two graphs the same length? Prove your answer.

4) Suppose that $y' = y + \cos x$ and $y(0) = 0$

a) Use Euler's method with $h = 0.1$ to estimate $y(0.3)$

b) Solve the differential equation, then find $y(0.3)$.

c) Find the relative error for part (a).

Math 3B Problem Set 4 Polar and Parametric Functions

1) Consider the function represented in polar form by $r = \dfrac{3}{2 - \cos\theta}$ and in parametric form by

$x = t + 1,\ \ y^2 = \dfrac{12 - 3t^2}{4}$.

a) Find the values of θ and *t* so that the two graphs are exactly the same.

b) Convert the polar form of the function to rectangular form. What shape is it?

c) Convert the parametric form of the function to rectangular form. Look at the answer to part (b) first.

d) Set up the integral that could be used to find the arc length of the curve.

e) Estimate the arc length of the curve.

2) Can a function in polar form have a tangent at a pole? Obviously, the function itself can pass through the pole at (0,0) But what has to happen to the derivative at this point?

I. Consider the function $r = \cos(2\theta)$ on $[0, 2\pi)$

a) Find the general equation that gives the slope of the tangent line.

b) Find both *r* and $\frac{dr}{d\theta}$ at $\frac{\pi}{4}$.

c) Find the equation(s) of the tangent line(s) to the function at the pole.

d) For what values of θ does $r = \cos(2\theta)$ have a tangent line at the pole?

II. Consider the function $r = 1 + \sin(2\theta)$ on $[0, 2\pi)$

a) Find the general equation that gives the slope of the tangent line.

b) Find both *r* and $\frac{dr}{d\theta}$ at $\frac{3\pi}{4}$.

c) Find the equation(s) of the tangent line(s) to the function at the pole.

d) For what values of θ does $r = 1 + \sin(2\theta)$ have a tangent line at the pole?

III. When will a function have a tangent at the pole? Explain what has to be true about the function *r* and the derivative $\frac{dr}{d\theta}$. What is the difference between $\frac{dr}{d\theta}$ and $\frac{dy}{dx}$ at this point?

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Math 3B Problem Set 5 Sequences Name________________________

1) Compare the sequences: $a_n = \frac{1}{n}$ and $b_n = \frac{1}{n^2}$ by answering questions a to i.

a) Find the first 10 terms of each sequence. Which sequence has smaller values?

b) Find the limit as $n \to \infty$ for each sequence. How do they compare?

c) Which of the two sequences goes to its respective limit faster?
How could you prove your answer?

d) Now use function to compare these sequences, let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$.
Graph both of these on one set of axes. Which one is smaller?

e) Find and compare the values of $\int_1^\infty f(x)\,dx$ and $\int_1^\infty g(x)\,dx$.

f) Find the sum of the first 10 terms of each sequence. (Use sum seq on the calculator)

g) Find the sum of the first 100 terms of each sequence.

h) Find the sum of the first 1000 terms of each sequence.

i) How much would each sum be if you added up ALL of the terms from 1 to infinity?

$\sum_{n=1}^{\infty} a_n =$

$\sum_{n=1}^{\infty} b_n$

2) Now compare the sequences: $a_n = \frac{1}{n^{0.9}}$ and $b_n = \frac{1}{n^{1.1}}$.

Use what you learned from the previous problem to evaluate each sum below:

$\sum_{n=1}^{\infty} a_n =$

$\sum_{n=1}^{\infty} b_n$

1) Show that the series $\sum \frac{1}{n^2+2n+1}$ converges.

2) Show that the series $\sum \frac{1}{n^2+2n+1}$ converges using a different method than above.

3) Show that the series $\sum \left(\frac{1}{n^2+2n+1}\right)^{\frac{3}{2}}$ converges.

4) Does the series $\sum \frac{1}{\sqrt{n^2+2n+1}}$ converge? Prove your answer.

5) Show that the series $\sum \frac{1}{n!}$ converges by comparing with the series $\sum \frac{1}{2^n}$.

6) Show that the series $\sum \frac{1}{n^n}$ converges.

7) Does the series $\sum \frac{3^n}{n!}$ converge? Prove your answer.

8) Does the series $\sum \frac{n!}{n^n}$ converge? Prove your answer.

Math 3B Problem Set 7 Series II Name____________________

1) Consider the series $\sum_{n=0}^{\infty} x^n$.

a) For what values of *x* will the series converge?

b) Find the sum of the series.

c) Use part (b) to find a series whose sum is $\frac{1}{x}$.

d) Find a series whose sum is $\ln x$.

2) Consider the series $\sum_{n=0}^{\infty} \frac{1}{n!}$. (Note: 0! = 1)

a) Prove that the series converges.

b) Estimate the series, accurate to 3 decimal places.

c) For what values of *x* will the $\sum_{n=0}^{\infty} \frac{x}{n!}$ converge?

d) Find the sum of the series $\sum_{n=0}^{\infty} \frac{x}{n!}$.

3) A biologist studied frogs at one particular lake in Canada. She found that from one year to the next only 8% of the previous years frogs remain and that each year there are 900 new frogs (either newborns or migrants) at this lake.

a) Assume there are 900 frogs at the lake in year 1. Find the number of frogs for years 2, 3 & 4.

b) What type of sequence is this?

c) Find a formula for the number of frogs at the lake during year *n.*

d) The ecosystem at this lake can not sustain more than 1000 frogs. Will there ever be a year when there will be more than 1000 frogs? Prove your answer.

4) Suppose you wanted to be able to give $1000 per year to the Hartnell College Scholarship Fund. Assume that you can invest a large chunk of money at 8% annually to cover this gift.

a) How much would the n^{th} payment be worth after *n* years?

b) How much would you have to invest today in order to cover the $1000 payment in *n* years?

c) Construct an infinite series that gives the amount you must invest today to cover all payments, forever.

d) Show that the series in part(c) converges and find the sum. What does this sum represent?

Math 3B Problem Set 8 Series Tests Name__________________

Test the series for convergence or divergence. Indicate which test you used. All series begin with $n=1$.

1) $\sum \frac{n-1}{n^2+n}$

2) $\sum \frac{1}{\sqrt{n^2+n}}$

3) $\sum \cos n$

4) $\sum \cos(\frac{\pi}{n})$

5) $\sum \frac{\tan(\frac{1}{n})}{n}$

6) $\sum \frac{\tan(\frac{1}{n})}{n^2}$

7) $\sum \frac{(-1)^n n}{n^2 + 3n + 1}$

8) $\sum \frac{(-1)^n \sqrt{n}}{n+5}$

9) $\sum \frac{e^{\frac{1}{n}}}{n^2}$

10) $\sum \frac{e^n}{n!}$

11) $\sum \frac{2^n n}{n!}$

12) $\sum 2^{\frac{1}{n}} - 1$

Math 3B Problem Set 9

1) For each sequence a_n below, answer questions (a) – (f).

a) Find $\lim_{n\to\infty} a_n$

b) $\int_1^\infty a_n$

c) Find $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$

d) Is a_n increasing or decreasing? Prove your answer.

e) Is $a_n \le \frac{1}{n^2}$? For which values of *n*?

f) Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge?

$$a_n = 3(\tfrac{1}{2})^n \qquad a_n = \frac{n}{n^2+1} \qquad a_n = \frac{e^{\frac{1}{n}}}{n^2}$$

2) Find the interval of convergence for each series below.

a) $\sum \frac{5x^n}{n+1}$

b) $\sum \frac{n^2}{2^n} x^n$

c) $\sum \frac{n!}{e^n} x^n$

Extra Credit!

Does the series $\sum \frac{\sin(2n)}{n}$ converge or diverge? Prove your answer.

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Math 3B Problem Set 10 Final Review

1) On the same coordinate system, graph $y = \sin^2 x$ and the first three Taylor polynomials for $y = \sin^2 x$ centered at $\frac{\pi}{2}$. You may use the fact that $\sin(2x) = 2\sin x \cos x$.

Use the tracing function of the calculator to estimate the error of $P_3(\frac{\pi}{3})$

2) Answer the following questions about $r = \sin^{-1}\theta$.

a) Is the graph of this function part of a parabola? Prove your answer.

b) Prove that $\frac{dr}{d\theta}$ does NOT equal 0 for any value of θ. Why is this?

c) Find the slope of the function at the pole.

d) Find the value of $\frac{dr}{d\theta}$ where $r = 0$.

e) Set up the integral to find arc length. Estimate the length.

f) Covert *r* to rectangular form.

3) Determine if the series converge or diverge. Tell which test you used and prove your answer.

a) $\sum \frac{\tan^2 n}{n^2}$ b) $\sum \frac{\ln n}{n^4}$ c) $\sum \frac{n^3 + 2^n}{n^2 + 3^n}$ d) $\sum \frac{(\ln n)^n}{n^n}$

4) Solve the differential equations with the given initial conditions.

a) $1 + x = xyy', \quad y(1) = -2$ b) $1 + xy = xy', \quad y(1) = -2$

5) Evaluate each integral. Show all work.

a) $\int \frac{e^x}{1 + e^x}\, dx$ b) $\int \frac{4}{x^2 - 1}\, dx$ c) $\int x\sqrt{x^2 - 9}\, dx$ d) $\int_0^1 \frac{\ln x}{\sqrt{x}}\, dx$

6) Find the centroid for the region that is bounded by $y = \sin x$ and $y = \cos x$ on the interval $[0, \frac{\pi}{4}]$.