Math 4 – Study Guide for Test 1 on Chapters 1 and 2

The test is in class on Monday, October 10, 2016. It will last 50 minutes. You’ll be able to use a calculator, and you can use a 3x5 card with handwritten notes on one or both sides (so up to 30 square inches of notes).

Due to the limited time, there will be just one problem that requires row operations to be written out. It might involve solving a linear system or calculating the inverse of a matrix. (You won’t be required to calculate a determinant using row operations, but depending on the matrix it could still be useful to do so.)

Here are some suggested exercises from Anton & Rorres, 11th edition, chapters 1 and 2, some of which have been assigned as homework:

Section 1.1: 1 – 15 odd, 19, 21, all true/false questions
Section 1.2: 1 – 19 odd, 37, all true/false questions
Section 1.3: 1 – 7 odd, 11, 13, 15, 23
Section 1.4: 1 – 19 odd, true/false (a), (d), (f), (j), (k)
Section 1.5: 11 – 21 odd
Section 1.6: 1 – 15 odd, all true/false questions except (e)
Section 1.7: 1 – 27 odd,
Section 2.1: 1 – 23 odd, 27, 29, 33
Section 2.2: 1 – 7 odd, 15, 17, 19, 21, 23, 25, all true/false questions except (c)
Section 2.3: 1, 3, 5, 7, 15, 17, 33, 35, true/false (a) through (d)

Sources for additional review problems:

- The take-home quiz we had on Chapter 1. If you’d like a clean copy, it’s on my web page (hartnell.edu/jriley); just scroll down to the link under Math 4 that says “Take-home quiz due Monday, September 19”.
- Chapter 1 Supplementary Exercises in the book, numbers 1 through 8.
- Chapter 2 Supplementary Exercises in the book, numbers 1 through 16 and number 27.

Here are some other problems from tests and quizzes I’ve given in past Linear Algebra classes:

1. Calculate

\[
\begin{bmatrix}
1 & 4 & 5 \\
3 & -2 & 6 \\
0 & 1 & -4
\end{bmatrix}
- \begin{bmatrix}
-3 & -5 & 0 \\
-8 & -3 & 4 \\
5 & 6 & 1
\end{bmatrix}
\]

2. Calculate

\[
\begin{bmatrix}
1 & 2 & 5 & 4 \\
4 & -7 & 1 & 6
\end{bmatrix}
\begin{bmatrix}
0 & 2 & 5 \\
1 & 0 & 0
\end{bmatrix}
- \begin{bmatrix}
-6 & 3 & 4
\end{bmatrix}
\]

3. Find the transpose of

\[
\begin{bmatrix}
4 & -7 & 3 & 2 \\
1.3 & 5 & 6 & -11 \\
0 & 1 & 4 & -3 \\
2 & 9 & 8 & -1
\end{bmatrix}
\]

On problems 4 and 5, solve the linear system by putting the augmented matrix in row echelon form, without using a calculator. (You can take it all the way to reduced row echelon form, but you don’t have to.) If there are infinitely many solutions, express the solution set in parametric form and state one specific solution. (For example, if the solution is \(x = 2t, y = 3t - 1, z = t\), one specific solution would be (2, 2, 1).)

4. \[
\begin{align*}
x - y + z &= 5 \\
2x + 3y &= 6 \\
2x - z &= -4
\end{align*}
\]

5. \[
\begin{align*}
x_1 - 3x_2 + 4x_3 + 7x_4 &= 3 \\
x_2 + 2x_3 + 2x_4 &= 2 \\
3x_1 - 9x_2 + 13x_3 + 26x_4 &= -8
\end{align*}
\]
6. Calculate the inverse of \[
\begin{bmatrix}
-2 & 0 & 1 \\
0 & -1 & -1 \\
1 & 1 & -4
\end{bmatrix}
\] without using a calculator.

7. Solve the matrix equation \[
\begin{bmatrix}
2 & 0 & 1 \\
0 & 1 & 1 \\
1 & 1 & -4
\end{bmatrix}
X = \begin{bmatrix}
9 \\
9 \\
-18
\end{bmatrix}
\]. The result of problem 6 may be useful.

8. Calculate the determinants \[
\begin{bmatrix}
2 & 3 & 4 & 5 \\
0 & 1 & 0 & 3 \\
0 & -2 & 5 & 7 \\
0 & -3 & 2 & 10
\end{bmatrix}
\] and \[
\begin{bmatrix}
2 & 0 & 0 & 0 \\
3 & 1 & 2 & 3 \\
4 & 0 & 5 & 2 \\
5 & 3 & 7 & 10
\end{bmatrix}
\] without using a calculator. (Hint: If you find yourself calculating the second or third determinant from scratch, you’re working harder than you need to.)

9. Using the fact that \[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & -7 & 9 & 8 \\
2 & 4 & -11 & 9 \\
21 & 3 & 14 & -1
\end{bmatrix}
\] \[
= -17208,
\] and \[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
15 & 3 & 19 & 18 \\
2 & 4 & -11 & 9 \\
21 & 3 & 14 & -1
\end{bmatrix}
\] \[
= -11478,
\] find \[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
10 & 10 & 10 & 10 \\
2 & 4 & -11 & 9 \\
21 & 3 & 14 & -1
\end{bmatrix}
\] without using a calculator. (Hint: use Theorem 2.3.1.)

10. Give an example of a single 2 x 2 matrix \(A\) that has all three of these properties: \(A\) is symmetric, the sum of the entries on the main diagonal of \(A\) is 0, and \(\det(A) = -4\).

11. (a) If \(A\) and \(B\) are invertible \(n \times n\) matrices, and if \(AB = BA\), explain why \(A^{-1}B^{-1} = B^{-1}A^{-1}\).

(b) If \(A\) is an invertible matrix such that \(A = A^{-1}\), explain why \(\det(A)\) must be either 1 or \(-1\).

12. In each part of this problem, give the dimensions that a matrix \(A\) must have if it meets the stated condition.

(a) \(A^T\) (the transpose of \(A\)) is a 4x2 matrix.

(b) \(A = BC\), where \(B\) is a 12x7 matrix and \(C\) is a 7x16 matrix.

(c) If \(D\) is a 14x8 matrix and \(E\) is an 11x15 matrix, then the products \(DA\) and \(AE\) both exist.

(d) \(\det(2A) = 8\det(A)\).

13. (a) Find the determinant of \[
\begin{bmatrix}
1 & 3 & -4 & 5 \\
0 & 2 & 6 & 8 \\
0 & 0 & 1 & 11 \\
0 & 0 & 0 & 1
\end{bmatrix}
\].

(b) If \(A\), \(B\), and \(C\) are invertible 4x4 matrices, with \(\det(A) = 1\), \(\det(B) = 2\), and \(\det(C) = 3\), then find \(\det(AB^{-1}C^T)\). Feel free to reuse your answer from part (a)!