

Adding and Subtracting Positive and Negative Numbers

I. Before we get started...

A *negative number* is a number that's less than 0, such as -3 or -10 . A negative number is always written with a $-$ sign. In words, we call a $-$ sign a "minus sign" or "negative sign." The $-$ sign is also used to indicate subtraction, as in $10 - 5$.

A *positive number* is a number that's greater than 0, such as 3 or 10. A positive number can be written with a $+$ sign in front of it (for example, we can write 3 as $+3$), but we usually only do that if we really want to emphasize that the number is positive. In words, we call a $+$ sign a "plus sign" or "positive sign." The $+$ sign is also used to indicate addition, as in $10 + 5$.

We use parentheses to avoid writing having two $+$ or $-$ signs in a row. For example, we write $3 + (-7)$ instead of $3 + -7$, and we write $4 - (-6)$ instead of $4 - -6$.

Don't let parentheses trick you into multiplying when you should be adding or subtracting:

$3 + (-7)$ means 3 *plus* -7 because there's a $+$ sign before the parentheses
 $3 (-7)$ means 3 *multiplied by* -7 because there's no operation sign (that is, no $+$ sign or $-$ sign) before the parentheses

When you add 0 to a number, or subtract 0 from a number, the answer is the same as the number you started with:

$3 + 0 = 3$ $0 + 4 = 4$
 $5 - 0 = 5$ $0 + (-7) = -7$
 $-4 + 0 = -4$

II. Adding two positive numbers

This is easy – just add the two numbers the same way you always have!

Example:

$$2 + 3 = 5$$

Exercises:

- $2 + 4 =$ _____
- $4 + 2 =$ _____
- $4 + 5 =$ _____
- $6 + 5 =$ _____
- $7 + 2 =$ _____
- $10 + 16 =$ _____
- $4 + 0 =$ _____

III. Adding two negative numbers

Step 1: Ignore the minus signs and add the two numbers as if they were both positive.

Step 2: Put a – sign in front of the answer you got in Step 1.

Note: When you add two negative numbers, the answer is always negative.

Example 1:

$-2 + (-3)$ First add 2 and 3, which gives you 5. Put a – sign before the 5.
The answer is -5 .

Example 2:

$-4 + (-5)$ First add 4 and 5, which gives you 9. Put a – sign before the 9. The answer is -9 .

Exercises:

1. $-2 + (-4) =$ _____ 2. $-3 + (-5) =$ _____ 3. $-9 + (-6) =$ _____
4. $-6 + (-5) =$ _____ 5. $-7 + (-2) =$ _____ 6. $-10 + (-16) =$ _____

IV. Adding a positive number and a negative number

First, we need to take a short detour and talk about the *absolute value* of a number.

The absolute value of a positive number is just the number. For example, the absolute value of 5 is 5. This also holds for 0: the absolute value of 0 is 0.

The absolute value of a negative number is the number you get by throwing away the minus sign. For example, the absolute value of -4 is 4.

We write the absolute value of a number by putting it between vertical bars: the absolute value of 5 is written as $|5|$, the absolute value of -4 is written as $|-4|$, and so on. So you can say things like this:

$$|5| = 5$$
$$|-4| = 4$$

Now, here's how to add a positive number and negative number:

Step 1: If the two numbers are opposites of each other, like 3 and -3 , then they add up to 0, so that's the answer and you can skip the remaining steps.

Step 2: Find the absolute value of both numbers, which gives you two positive numbers.

Step 3: Subtract the smaller of these two positive numbers from the larger one.

Step 4: If the negative number in the original problem has the larger absolute value of the two, put a minus sign before the answer you got in Step 3. Otherwise, leave your answer from Step 3 alone. You're done!

Note: When you add a positive number and a negative number, the answer can be positive or negative:

If the **negative** number has the larger absolute value, then the answer is **negative**.

If the **positive** number has the larger absolute value, then the answer is **positive**.

Example 1:

$4 + (-4)$ Step 1: 4 and -4 are opposites of each other, so they add up to 0, and we can skip the remaining steps. **The answer is 0.**

Example 2:

$7 + (-3)$ Step 1: 7 and -3 aren't opposites of each other, so go on to Step 2.
Step 2: $|7| = 7$ and $|-3| = 3$. Our two positive numbers are therefore 7 and 3.
Step 3: Since 3 is smaller than 7, subtract 3 from 7: $7 - 3 = 4$
Step 3: $|7|$ is greater than $|-3|$ (that's just saying 7 is greater than 3), so you leave the answer from Step 3 alone. **The answer is 4.**

Example 3:

$(-5) + 2$ Step 1: -5 and 2 aren't opposite of each other, so go on to Step 2.
Step 2: $|-5| = 5$ and $|2| = 2$. Our two positive numbers are therefore 5 and 2.
Step 3: Since 2 is smaller than 5, you subtract 2 from 5: $5 - 2 = 3$
Step 3: $|-5|$ is greater than $|2|$ (that's just saying 5 is greater than 2), so you put a minus sign in front of the answer from Step 3. **The answer is -3 .**

Exercises:

1. $6 + (-6) =$ _____
2. $10 + (-7) =$ _____
3. $-3 + 5 =$ _____
4. $-9 + 6 =$ _____
5. $5 + (-5) =$ _____
6. $-7 + 8 =$ _____
7. $8 + (-7) =$ _____
8. $10 + (-14) =$ _____
9. $-14 + 10 =$ _____
10. $-4 + 0 =$ _____

V. Subtracting

First off, you're probably already comfortable with subtracting a smaller positive number from a larger positive number. Here are some examples:

$$7 - 3 = 4$$

$$10 - 1 = 9$$

$$18 - 11 = 7$$

Those work the same way they always have, and we've already been doing subtractions like those in section IV above.

To solve other subtraction problems, we turn them into addition problems. You can always turn a subtraction problem into an addition problem, because *subtracting a number is the same as adding its opposite*. For example, subtracting 3 is the same as adding -3 , and subtracting -4 is the same as adding 4 (since 4 is the opposite of -4).

We already know how to add every combination of positive and negative numbers, so that means we know how to subtract them too.

Example 1:

$-4 - 5$ This is the same as $-4 + (-5)$, which we already solved as Example 2 under “II. Adding two negative numbers.” **The answer is -9 .**

Example 2:

$2 - 5$ This is the same as $2 + (-5)$, which we already solved as Example 3 under “III. Adding a positive number and a negative number.” At first glance, it doesn’t look like the same problem, because that example was $-5 + 2$. However, the Commutative Property of Addition tells us $2 + (-5)$ is the same as $-5 + 2$, so the answer is the same either way: -3 .

Example 3:

$4 - (-3)$ This is the same as $4 + 3$ (because 3 is the opposite of -3). **The answer is 7.**

Note: When people see a problem like $4 - (-3)$ in Example 3 above, they often want to turn it into $4 - 3$ instead of $4 + 3$. That gives the wrong answer, though, so don’t do that!

Exercises:

1. $6 - 5 =$ _____
2. $6 - 6 =$ _____
3. $6 - 7 =$ _____
4. $7 - 2 =$ _____
5. $7 - (-2) =$ _____
6. $-2 - 7 =$ _____
7. $0 - 5 =$ _____
8. $-3 - (-6) =$ _____
9. $(-5) - (-4) =$ _____

Answers to the Exercises

Section II

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|-------|------|-------|
| 1. 6 | 2. 6 | 3. 9 |
| 4. 11 | 5. 9 | 6. 26 |
| 7. 4 | | |

Section III

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|--------|-------|--------|
| 1. -6 | 2. -8 | 3. -15 |
| 4. -11 | 5. -9 | 6. -26 |

Section IV

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|--------|-------|-------|
| 1. 0 | 2. 3 | 3. 2 |
| 4. -3 | 5. 0 | 6. 1 |
| 7. 1 | 8. -4 | 9. -4 |
| 10. -4 | | |

Section V

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|-------|------|-------|
| 1. 1 | 2. 0 | 3. -1 |
| 4. 5 | 5. 9 | 6. -9 |
| 7. -5 | 8. 3 | 9. -1 |