

Motion in One Dimension with Air Drag

Purpose: To create a spreadsheet that calculates position and velocity data as a function of time for one dimensional vertical motion in the presence of air drag.
To use this data to create graphs of velocity and position versus time.

Introduction:

The future position and velocity of an object can be calculated based on the present position and velocity if the forces acting on the object are known. Knowledge of the forces at some point in time allows one to find the acceleration at that time. From the acceleration a future velocity can be predicted and knowing the velocity one can predict the position at some future time. The accuracy of the method depends on the method of calculation and the size of the time interval chosen. The method of calculation to be used involves finding a “half step” velocity and acceleration which will be explained by your instructor.

Essentially, we try to predict a new velocity, v_{new} , and new position, y_{new} , by the following motion equations

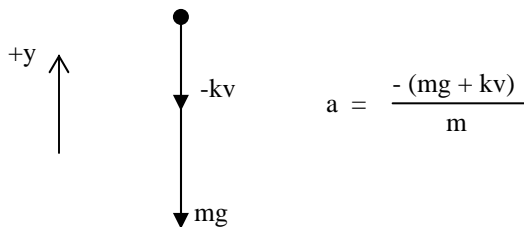
$$\begin{aligned}v_{\text{new}} &= v_{\text{old}} + a_{\text{avg}}\Delta t, \\y_{\text{new}} &= y_{\text{old}} + v_{\text{avg}}\Delta t\end{aligned}$$

where v_{old} and y_{old} are the previous values of velocity and position, and a_{avg} and v_{avg} are the average values of the acceleration and velocity during the time interval Δt . This is only an approximation since the acceleration and velocity are changing during the time interval, but we can obtain accurate results if we use a small Δt and make a wise choice for the average velocity and acceleration. Your instructor will show you an interesting way to make this choice.

Since the acceleration depends on the forces acting on the object, we must specify precisely what these forces are. In our problem the drag force will at first be assumed to depend on the first power of the velocity and can be written as

$$F_D = -kv$$

where k is a proportionality constant. Thus the acceleration, a , of an object of mass, m , moving upward through the air under the influence of both the force of gravity and air drag is



Eventually the object will reach its maximum height ($v = 0$) and will accelerate downward. At some later time, it will reach a terminal speed v_t . When this occurs, the acceleration is zero and the force of gravity and the drag force just cancel each other. Therefore, the magnitude of F_D and the weight are the same:

$$kv_t = mg$$

and therefore

$$k = mg/v_t$$

Now, the expression for the acceleration above can be rewritten as:

$$1) \quad a = -g(1 + v/v_t).$$

The nice thing about this expression is that it is valid for when the object is moving upward ($v > 0$) and when it is moving downward ($v < 0$).

Equipment: Computer with excel and graphical analysis software.

Procedure:

1. Set up a spreadsheet in **Excel** for calculating the one dimensional motion of an object subject to a drag force as given above. Assume the object is thrown initially upward with a speed of 100 m/s, that the initial height is 1000 m and the terminal speed of the object is 40 m/s. Use the formula in equation 1 above to determine the acceleration of the object. Follow the example spreadsheet attached to this procedure using a time interval of 0.1 s. Your instructor will explain the method of calculating the half step velocity and acceleration. Have the spreadsheet calculate the motion for the first 20 s.
2. Verify with your instructor that your spreadsheet is working properly and then **Copy** your data (time, position, and velocity columns) from the spreadsheet and **Paste** it into the **Graphical Analysis** program so that you can construct graphs of position vs time and velocity vs time. You need to make room for the data in Graphical Analysis by creating a new column for the velocity data. Do this by choosing **Data/New Manual Column** entered from the menu bar in Graphical Analysis. Name the position data column y1 and the velocity data column v1. Supply the appropriate units for each column. Create two side by side windows and plot position vs time in one and velocity vs time in the other. Double click on each graph and turn off the point protectors to improve the appearance of the graphs. Verify with your instructor that your graphs show the correct variation of position and velocity with time. Put titles on your graphs and be sure the axes are labeled properly.
3. Repeat step 1 above but this time we will look at a drag force that is dependent on the square of the velocity. Assuming a drag force, $F_D = kv^2$, determine a formula for calculating the acceleration in a similar way that was done for equation 1. Use the same columns in your spreadsheet that you used in step one but with the modified calculation for the acceleration column.
4. Repeat step 2 above. Name the new columns created in Graphical Analysis, y2 and v2 for the position and velocity data columns. Place both position graphs (y1 and y2) on the same graph and both velocity graphs (v1 and v2) on the same graph—two graphs side by side.
5. Finally, create a spreadsheet that calculates positions and velocities for a falling body with no drag force (freely falling). Use the same initial conditions as for the other two cases. Use the same columns again but with a different (and much simpler) acceleration calculation. In Graphical Analysis, create two columns, named y0 and v0, copy and paste your spreadsheet data into these columns, and plot this data onto the two graphs created in step 4 above. You should now have three curves plotted on each graph. Print out the two graphs (one page) and label clearly with a pen or pencil each curve with the drag force used for its calculation.
6. Your lab report should include a printout (first page only!) of the spreadsheet calculations for the drag force used in part 3 above. Also make a table showing the first two rows of formulae used in each of the three spreadsheets (no drag force, $F_D \propto v$, and $F_D \propto v^2$) that you created.

Falling Bodies in One Dimension with Air Drag

Drag Force: $F_d = -kv$, $k = mg/v_t$

(Sample Data)

Initial Values

(at $t = 0$)

time =	0	s
g =	9.8	m/s ²
velocity =	100	m/s
position =	1000	m
terminal speed (v_t) =	40	m/s
time step (Δt) =	0.1	s

time (s)	y (m)	v (m/s)	a (m/s ²)	v-halfstep	a-halfstep
0	1000	100	-34.3	98.285	-33.879825
0.1	1009.829	96.612018	-33.46994429	94.93852	-33.05993747
0.2	1019.322	93.306024	-32.65997582	91.673025	-32.25989112
0.3	1028.49	90.080035	-31.86960849	88.486554	-31.47920578
0.4	1037.338	86.932114	-31.09836795	85.377196	-30.71741294
0.5	1045.876	83.860373	-30.34579133	82.343083	-29.97405538
0.6	1054.11	80.862967	-29.61142697	79.382396	-29.24868699
0.7	1062.049	77.938099	-28.89483414	76.493357	-28.54087242
0.8	1069.698	75.084011	-28.19558277	73.674232	-27.85018688
0.9	1077.065	72.298993	-27.51325319	70.92333	-27.17621584
1	1084.158	69.581371	-26.8474359	68.238999	-26.51855481
1.1	1090.982	66.929516	-26.19773131	65.619629	-25.8768091
1.2	1097.544	64.341835	-25.56374948	63.063647	-25.25059355
1.3	1103.85	61.816775	-24.94510994	60.56952	-24.63953234
1.4	1109.907	59.352822	-24.34144414	58.13575	-24.04325874
1.5	1115.72	56.948496	-23.75238156	55.760877	-23.46141489
1.6	1121.297	54.602355	-23.1775769	53.443476	-22.89365158
1.7	1126.641	52.31299	-22.61668243	51.182155	-22.33962807
1.8	1131.759	50.079027	-22.06936154	48.975559	-21.79901187
1.9	1136.657	47.899126	-21.53528575	46.822361	-21.2714785
2	1141.339	45.771978	-21.01413453	44.721271	-20.75671138
2.1	1145.811	43.696307	-20.5055951	42.671027	-20.25440156
2.2	1150.078	41.670866	-20.00936226	40.670398	-19.76424758
2.3	1154.145	39.694442	-19.5251382	38.718185	-19.28595525
2.4	1158.017	37.765846	-19.05263229	36.813214	-18.81923755
2.5	1161.698	35.883922	-18.59156097	34.954344	-18.36381435
2.6	1165.194	34.047541	-18.14164752	33.140459	-17.91941234
2.7	1168.508	32.2556	-17.70262192	31.370469	-17.4857648
2.8	1171.645	30.507023	-17.27422068	29.643312	-17.06261148
2.9	1174.609	28.800762	-16.8561867	27.957953	-16.64969841
3	1177.405	27.135792	-16.44826909	26.313379	-16.24677779
3.1	1180.036	25.511114	-16.05022303	24.708603	-15.8536078